Plain language summary

Explainer
Developing maths proficiency

October 2024

Maths permeates and influences everyday life. Proficiency in maths is fundamental to a student's achievement at school and beyond. This explainer outlines how maths proficiency develops and the policies and practices that support effective learning.

Developing maths proficiency involves students having the skills, knowledge and dispositions to apply their mathematical knowledge across a range of contexts. Early maths proficiency is strongly linked with achieving academic success (Byun et al., 2015; Davis-Kean et al., 2022; You & Nguyen, 2012) and obtaining lifelong personal and professional opportunities (Codding et al., 2016; Ritchie & Bates, 2013). It contributes to Australia's education goals of developing students who can engage in the world as active and informed citizens (Crowe, 2010; Council of Australian Governments Education Council, 2019).

This explainer complements AERO's model of learning and teaching, which identifies the most effective and efficient teaching practices aligned with how students learn. The model can be used by teachers, school leaders and policymakers to recognise and focus on the practices and policies that will most effectively support students' developing maths proficiency. This explainer defines and details the importance of the 5 different strands of maths proficiency and how effective and efficient teaching can support their development. Teaching practices that align with how students learn will improve learning outcomes for students across all learning areas. By sequencing learning tasks, as well as breaking up and explicitly teaching mathematical content, teachers can help students' proficiency to grow. This, in turn, supports positive dispositions.

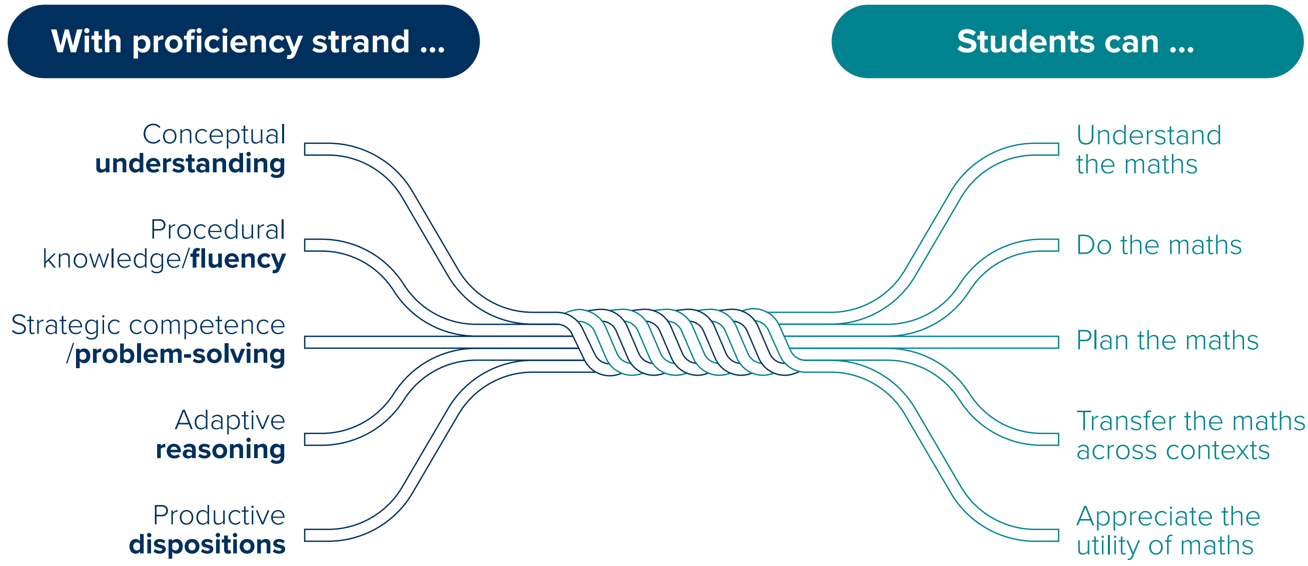
Components of maths proficiency

Maths proficiency strands

Maths proficiency is comprised of multiple strands, which are interrelated, develop simultaneously and are mutually reinforcing (Kilpatrick et al., 2001). The Australian Curriculum recognises the proficiency strands of understanding, fluency, problem-solving and reasoning in mathematics (Australian Curriculum, Assessment and Reporting Authority [ACARA], 2024a). The Australian Curriculum also recognises numeracy as a General Capability, highlighting the importance of students' appreciation of maths, particularly as it applies to real-world contexts (ACARA, 2024b).

This informs students' dispositions, which – alongside knowledge, skills and behaviours – support proficiency (ACARA, 2024c). This explainer explores the Australian Curriculum proficiency strands alongside productive dispositions, as shown in Figure 1.

Figure 1: Maths proficiency strands



How maths proficiency develops

According to the Australian Curriculum, 'Students engage in mathematical problem solving when they are presented with a problem situation for which they do not immediately know the answer, and they work through a process of planning, applying strategies and heuristics to find a solution to the problem, reviewing and analysing their solution' (ACARA, 2024c). To successfully complete maths tasks and problems, students must leverage their **conceptual understanding** of a problem to select, plan and apply relevant strategies to do the maths, and follow the related **procedures** accurately.

Mathematical learning is cumulative (Jordan et al., 2010). Students who have fluent recall of knowledge and know how to apply it (**fluency**) are more likely to learn related complex concepts (Watts et al., 2014). The opposite is true for those lacking fluency (de Bruin et al., 2023). Fluent recall of relevant knowledge from long-term memory also supports **problem-solving** capability. However, attempts to build fluency in procedural knowledge without also addressing conceptual understanding are likely to result in misconceptions and students forgetting (Taylor & Kowalski, 2014).

Adaptive reasoning in maths requires students to apply mathematical skills and knowledge across a variety of contexts. This is referred to as 'transfer'. It also requires using logic to explain and justify why a particular approach was chosen (such as grouping 12 counters into 3 groups of 4 to demonstrate that 3 × 4 = 12). Students develop adaptive reasoning when they apply their knowledge and skills to varied and complex tasks. Varying the tasks to which students apply their growing knowledge and skills helps them recognise underlying structures and relationships between mathematical concepts so they can apply their developing maths proficiency in meaningful and useful ways (Dunlosky et al., 2013).

Dispositions towards maths, such as experiences of self-efficacy or anxiety towards maths, can influence students' motivation, engagement and achievement (Caviola et al., 2021; Fast et al., 2010; Hwang & Son, 2021). This then influences their persistence with difficult problems and the accuracy of their calculations (Barroso et al., 2021). Worries about maths can take up working memory. This limits students' ability to process information or retain learning and may also lead to them avoiding tasks (Beilock & Willingham, 2014; Gilmore et al., 2018).

How teaching can optimise maths proficiency

Promote early success in developing maths proficiency and intervene to address gaps or misconceptions

Early screening and timely maths intervention (such as through a multi-tiered system of supports framework) are critical to ensure all children can experience success and develop positive dispositions for learning right from the start. If secondary school students are identified as requiring additional support, then interventions can still be effective, but earlier intervention is preferable. Because of the cumulative nature of maths, earlier intervention can focus on fewer areas of maths and reduce the need for intervention in later grades.

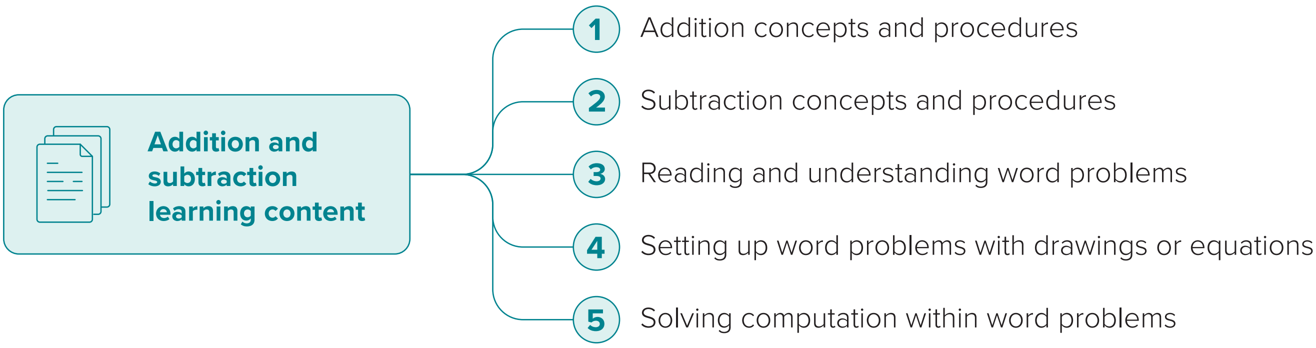
Sequence teaching to build complexity

The thinking required to solve a maths problem happens in working memory – which has limited capacity. Making sense of the task at hand, selecting potential strategies and then following procedures can very quickly lead to cognitive overload. This reduces a student's capacity to retain new learning in memory. Teaching that actively manages cognitive load has a positive impact on students' ability to engage with learning tasks and retain new learning.

Cognitive load is impacted by the volume of learning, but also by complexity. Learning must be sequenced to build in complexity over time. Planning experiences using a Concrete-Pictorial-Abstract (CPA) framework is one way to stage complexity in a way that supports students' understanding of mathematical concepts. Starting with concrete representations and gradually shifting to pictorial and then abstract representations can reduce complexity and prevent cognitive overload as students develop procedural knowledge and conceptual understanding (Warren & Miller, 2013; Uttal et al., 2013).

Breaking up the learning content and teaching chunks of new information explicitly with a modelled explanation aids fluency. For example, for primary students to develop fluency with addition and subtraction word problems, teachers need to break up and sequence the learning, as shown in Figure 2.

Figure 2: Example of breaking up and sequencing maths content



Provide opportunities to practise, develop and apply learning

Students more readily apply learning when they have regular opportunities to consolidate new knowledge and skills in long-term memory, such as through spacing and retrieval practices. For example, fluent recall of multiplication tables provides the foundation for more complex applications, such as identifying factors and simplifying fractions.

Once students have developed fluency and mastery over an area of learning, they benefit more from engaging in varied problem-solving tasks than from continued explicit demonstration and instruction (Kalyuga, 2007; Kirschner et al., 2006). This is known as the expertise reversal effect (Kalyuga et al., 2003). For example, once students know the names of different three-dimensional shapes and their features, they would then benefit from a task where they can apply their learning. Teachers might ask them to pick real three-dimensional objects that are commonly found in the classroom or at home and then identify all the faces, edges and bases. Opportunities for students to apply their mastery with greater independence to increasingly complex tasks can include providing guided, structured inquiry tasks, increasingly unfamiliar tasks, and authentic, contextual problems to solve (Martin & Evans, 2018).

Include resources and real-life examples to encourage proficiency

Supporting students to appreciate the utility of maths can improve proficiency. Teachers can play a role by highlighting the usefulness of maths in context, promoting the beauty of maths or identifying remarkable patterns, and providing opportunities for high levels of success (Rosenshine, 2012; Willingham, 2021). Opportunities to apply maths proficiency within real-world, relevant and context-specific learning experiences can aid the successful transfer of learning (Brown et al., 1989). For example, students might be presented with a task to identify the two-dimensional shapes that make up a section of the playground, and then must calculate the area and cost of turf required to fill it.

Real-world, relevant and context-specific examples can support students' ability to transfer their learning to future applications. Such opportunities support understanding of the patterns, relationships and underlying principles in that area of learning (Brown et al., 1989; Kirschner & Hendrick, 2020). For example, once a student has calculated the area and cost of turf for a few different playgrounds, they'll become more confident splitting up complex shapes, understand context-specific terms such as 'wastage', and be able to effectively apply similar principles to other contexts, such as painting the walls of a house. Such examples also impact students' sense of self and identity as confident and effective maths learners (Miller et al., 2023).

Promote oral and written language development together with maths proficiency

Early literacy skills are markers of future success in both literacy and numeracy (Jackson et al., 2023). Language and literacy skills – particularly understanding and using oral and written mathematical language – are critical for developing maths proficiency (Espinas & Fuchs, 2022; Riccomini et al., 2015; Warren & Miller, 2013). Reading and interpreting word problems place the brain under cognitive load before any maths has even taken place, especially if language contexts and examples aren't familiar or culturally relevant to students. Teachers should seek to develop students' skills in understanding and using mathematical language. Domain-specific mathematical language should be taught in the context of lessons, as worded problems may not unpack this language explicitly. Oral communication is also relevant to learning maths, including speaking and listening, as well as understanding vocabulary. Teaching mathematical language orally can be supported by using rich mathematical representations, including visual models and examples (McDonald et al., 2011; Miller & Armour, 2021).

Teachers need to be aware of potential differences in mathematical terms between Standard Australian English and other languages, including First Nations languages (Edmonds-Wathen et al., 2014). Students and their dispositions towards learning maths can benefit from using resources that start from a student's home language (which may or may not be Standard Australian English) and culture, and then make clear any explicit connections with mathematical terms (Jorgensen, 2015; Jorgensen [Zevenbergen], 2016; Miller & Armour, 2021). Barriers relating to language, as well as the context and culture of learning, can also be addressed by using culturally appropriate resources when learning maths (Miller et al., 2023). Scaffolds that provide access to meaning through multiple sources, including visual, aural and print sources, are most effective (Centre for Education Statistics and Evaluation, 2021). Mathematical tasks that are linked to a variety of real-world applications – and thinking aloud to make these connections transparent during teaching – can help students recognise applications and underlying patterns for the diverse ways they can apply mathematical procedures (Collins et al., 1991).

Implications for policy and practice

- Evidence-based teaching practices (such as those described in AERO's model of learning and teaching) are required in all maths classrooms to align with the processes of acquiring, retaining, retrieving and consolidating learning. Teachers need both general pedagogical knowledge and specific mathematical content knowledge to execute these practices effectively.
- Screening students early and often can ensure that students receive targeted and timely interventions. Early intervention and maths support is critical to ensure all children can experience success and thereby develop positive dispositions for learning.
- Curriculum development should sequence learning appropriate to the hierarchical, cumulative nature of maths. Teaching and learning plans should first explicitly teach related skills and knowledge and give opportunities to practise. Open and complex tasks with many possible paths to solutions should follow.
- Language requirements of mathematical tasks must be recognised, planned for, explicitly taught and, when students are ready, connected to real-world, relevant and context-specific learning opportunities. This should be reflected in curriculum resources and teaching and learning programs and materials.

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References

Australian Curriculum, Assessment and Reporting Authority. (2024a). *Mathematics proficiencies*. https://www.australiancurriculum.edu.au/resources/mathematics-proficiencies/

Australian Curriculum, Assessment and Reporting Authority. (2024b). *Numeracy*. https://v9.australiancurriculum.edu.au/f-10-curriculum.html/general-capabilities/numeracy?element=0&sub-element=0.

Australian Curriculum, Assessment and Reporting Authority. (2024c). *Numeracy (version 8.4)*. https://www.australiancurriculum.edu.au/f-10-curriculum/general-capabilities/numeracy/

Barroso, C., Ganley, C. M., McGraw, A. L., Geer, E. A., Hart, S. A., & Daucourt, M. C. (2021). A meta-analysis of the relation between math anxiety and math achievement. *Psychological Bulletin, 147*(2), 134. https://doi.org/10.1037/bul0000307

Beilock, S. L., & Willingham, D. T. (2014). Math anxiety: Can teachers help students reduce it? Ask the cognitive scientist. *American Educator, 38*(2), 28. https://www.aft.org/ae/summer2014/beilock_willingham

Brown, J., Collins, A., & Duguid, P. (1989). Situated cognition and the culture of learning. *Educational Researcher, 18*(1), 32–42.

Byun, S. Y., Irvin, M. J., & Bell, B. A. (2015). Advanced math course taking: Effects on math achievement and college enrollment. *The Journal of Experimental Education, 83*(4), 439–468. https://doi.org/10.1080/00220973.2014.919570

Caviola, S., Toffalini, E., Giofrè, D., Ruiz, J. M., Szűcs, D., & Mammarella, I. C. (2021). Math performance and academic anxiety forms, from sociodemographic to cognitive aspects: A meta-analysis on 906,311 participants. *Educational Psychology Review*, 1–37. https://doi.org/10.1007/s10648-021-09618-5

Centre for Education Statistics and Evaluation. (2021). *English as an additional language or dialect (EAL/D) effective school practices: Research report*. NSW Department of Education. https://education.nsw.gov.au/about-us/education-data-and-research/cese/publications/research-reports/eald-effective-school-practices

Codding, R. S., Volpe, R. J., & Poncy, B. C. (2016). *Effective math interventions: A guide to improving whole-number knowledge*. Guilford Publications.

Collins, A., Brown, J. S., & Holum, A. (1991). Cognitive apprenticeship: Making thinking visible. *American Educator, 15*(3), 6–11, 38–46. https://www.aft.org/ae/winter1991/collins_brown_holum

Council of Australian Governments Education Council. (2019). *The Alice Springs (Mparntwe) Education Declaration*. https://www.education.gov.au/alice-springs-mparntwe-education-declaration/resources/alice-springs-mparntwe-education-declaration

Crowe, A. R. (2010). "What's math got to do with it?": Numeracy and social studies education. *The Social Studies, 101*(3), 105–110. https://doi.org/10.1080/00377990903493846

Davis-Kean, P. E., Domina, T., Kuhfeld, M., Ellis, A., & Gershoff, E. T. (2022). It matters how you start: Early numeracy mastery predicts high school math course-taking and college attendance. *Infant and Child Development, 31*(2), e2281. https://doi.org/10.1002/icd.2281

de Bruin, K., Kestel, E., Francis, M., Forgasz, H., & Fries, R. (2023). *Supporting students significantly behind in literacy and numeracy: A review of evidence-based approaches.* Australian Education Research Organisation & Monash University. https://www.edresearch.edu.au/research/research-reports/supporting-students-significantly-behind-literacy-numeracy

Dunlosky, J., Rawson, K. A., Marsh, E. J., Nathan, M. J., & Willingham, D. T. (2013). Improving students' learning with effective learning techniques: Promising directions from cognitive and educational psychology. *Psychological Science in the Public Interest*, *14*(1), 4–58. https://doi.org/10.1177/1529100612453266

Edmonds-Wathen, C., Owens, K., Sakopa, P., & Bino, V. (2014). Indigenous languages and mathematics in elementary schools. In J. Anderson, M. Cavanagh, & A. Prescott (Eds.), *Proceedings of the 37th Annual Conference of the Mathematics Education Research Group of Australasia: Curriculum in Focus: Research Guided Practice* (pp. 207–214). Mathematics Education Research Group of Australasia.

Espinas, D. R., & Fuchs, L. S. (2022). The effects of language instruction on math development. *Child Development Perspectives*, *16*(2), 69–75. https://doi.org/10.1111/cdep.12444

Fast, L., Lewis, J., Bryant, M., Bocian, K., Cardullo, R., Rettig, M., & Hammond, K. (2010). Does math self-efficacy mediate the effect of the perceived classroom environment on standardized math test performance? *Journal of Educational Psychology*, *102*, 729–740. https://doi.org/10.1037/a0018863

Gilmore, C., Gobel, S.M., & Inglis, M. (2018). *An introduction to mathematical cognition.* Routledge.

Hwang, S., & Son, T. (2021). Students' attitude toward mathematics and its relationship with mathematics achievement. *Journal of Education and e-Learning Research, 8*(3), 272–280. http://asianonlinejournals.com/index.php/JEELR/article/view/3198

Jackson, C., Wan, W.-Y., Lee, E., Marslen, T., Lu, L., Williams, L., Collier, A., Johnston, K., & Thomas, M. (2023). *Which skills are important for future literacy and numeracy learning? How the Australian Early Development Census data reveal the building blocks for future reading, writing and numeracy performance*. Australian Education Research Organisation. https://www.edresearch.edu.au/research/research-reports/which-skills-are-important-future-literacy-and-numeracy

Jordan, N. C., Glutting, J., & Ramineni, C. (2010). The importance of number sense to mathematics achievement in first and third grades. *Learning and Individual Differences, 20*(2), 82–88. https://doi.org/10.1016/j.lindif.2009.07.004

Jorgensen, R. (2015). Language, culture and access to mathematics: A case of one remote Aboriginal community. *Intercultural Education*, *26*(4), 313–325. https://doi.org/10.1080/14675986.2015.1072302

Jorgensen (Zevenbergen), R. (2016). Playing the game of school mathematics: Being explicit for Indigenous learners and access to learning. *Intercultural Education*, *27*(4), 321–336. https://doi.org/10.1080/14675986.2016.1203586

Kalyuga, S. (2007). Expertise reversal effect and its implications for learner-tailored instruction. *Educational Psychology Review, 19*(4), 509–539. https://doi.org/10.1007/s10648-007-9054-3

Kalyuga, S., Ayres, P., Chandler, P., & Sweller, J. (2003). The expertise reversal effect. *Educational Psychologist*, *38*(1), 23–31. https://doi.org/10.1207/S15326985EP3801_4

Kilpatrick, J., Swafford, J., & Findell, B. (Eds.). (2001). *Adding it up: Helping children learn mathematics*. National Academy Press. https://nap.nationalacademies.org/catalog/9822/adding-it-up-helping-children-learn-mathematics

Kirschner, P. A., & Hendrick, C. (2020). *How learning happens: Seminal works in educational psychology and what they mean in practice*. Routledge. https://doi.org/10.4324/9780429061523

Kirschner, P. A., Sweller, J., & Clark, R. E. (2006). Why minimal guidance during instruction does not work: An analysis of the failure of constructivist, discovery, problem-based, experiential, and inquiry-based teaching. *Educational Psychologist, 41*(2), 75–86. https://doi.org/10.1207/s15326985ep4102_1

Martin, A., & Evans, P. (2018). Load reduction instruction: Exploring a framework that assesses explicit instruction through to independent learning. *Teaching and Teacher Education*, *73*. https://doi.org/10.1016/j.tate.2018.03.018

McDonald, S., Warren, E., & DeVries, E. (2011). Refocusing on oral language and rich representations to develop the early mathematical understandings of Indigenous students. *The Australian Journal of Indigenous Education*, *40*(1), 9–17. https://doi.org/10.1375/ajie.40.9

Miller, J., & Armour, D. (2021). Supporting successful outcomes in mathematics for Aboriginal and Torres Strait Islander students: A systematic review. *Asia-Pacific Journal of Teacher Education*, *49*(1), 61–77. https://doi.org/10.1080/1359866X.2019.1698711

Miller, J., Armour, D., Shay, M., & Sawatzki, C. (2023). What next? Building on the evidence of teaching and learning mathematics for Aboriginal and Torres Strait Islander students. In N. Moodie, K. Lowe, R. Dixon, & K. Trimmer (Eds.), *Assessing the evidence in Indigenous education research: Implications for policy and practice* (pp. 201–227). Springer International Publishing. https://doi.org/10.1007/978-3-031-14306-9_11

Riccomini, P. J., Smith, G. W., Hughes, E. M., & Fries, K. M. (2015). The language of mathematics: The importance of teaching and learning mathematical vocabulary. *Reading & Writing Quarterly*, *31*(3), 235–252. https://doi.org/10.1080/10573569.2015.1030995

Ritchie, S. J., & Bates, T. C. (2013). Enduring links from childhood mathematics and reading achievement to adult socioeconomic status. *Psychological Science, 24*(7), 1301–1308. https://doi.org/10.1177/0956797612466268

Rosenshine, B. (2012). Principles of instruction: Research-based strategies that all teachers should know. *American Educator*, *36*(1), 12–19. https://www.aft.org/ae/spring2012/rosenshine

Taylor, A. & Kowalski, P. (2014). Student misconceptions: Where do they come from and what can we do? In V. Benassi, C. Overson, & C. Hakala (Eds.), *Applying science of learning in education – Infusing psychological science into the curriculum* (pp. 259–273). American Psychological Association.

Uttal, D. H., Amaya, M., del Rosario Maita, M., Hand, L. L., Cohen, C. A., O'Doherty, K., & DeLoache, J. S. (2013). It works both ways: Transfer difficulties between manipulatives and written subtraction solutions. *Child Development Research, 2013*. https://doi.org/10.1155/2013/216367

Warren, E., & Miller, J. (2013). Young Australian Indigenous students' effective engagement in mathematics: The role of language, patterns, and structure. *Mathematics Education Research Journal*, *25*(1), 151–171. https://doi.org/10.1007/s13394-013-0068-5

Watts, T. W., Duncan, G. J., Siegler, R. S., & Davis-Kean, P. E. (2014). What's past is prologue: Relations between early mathematics knowledge and high school achievement. *Educational Researcher, 43*(7), 352–360. https://doi.org/10.3102/0013189X14553660

Willingham, D. T. (2021). *Why don't students like school? A cognitive scientist answers questions about how the mind works and what it means for the classroom* (2nd ed.). John Wiley & Sons.

You, S., & Nguyen, J. (2012). Multilevel analysis of student pathways to higher education. *Educational Psychology, 32*(7), 860–882. https://doi.org/10.1080/01443410.2012.746640

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